MATHEMATICS OF SCALE

(The end of boundary between Mathematics and Physic?)

By*:* Francisco de Asís Fernández Díaz

Abstract: The real number has an structure and properties beyond structure and properties of nature. But, this status is obviated in our use for to measure physic observables and how we know it generates many problems.

It presents here a number which properties approach those of the nature structure. That is to say, a purely physical number. Also it introduces an extension of the above mentioned numbers across what it are named numerical cycle-bases, structures that it extends the concept of imaginary number from a different perspective.

On the other hand, a few conceptual bases are created for the later establishment of a metric.

CHAPTER I

THE NATURAL OF SCALE

Introduction.

The origin of this approach is the incongruity that appears between Cosmos structure and real number use in its mathematical description..

The base of the mathematical development is in the arithmetic and in last instance in the number and the geometry.

The number was created to count in human scale. This way the so called natural numbers fulfill the count of units function perfectly and its addition (or multiplication that does not stop being a briefed sum).

Its extension - the integer numbers set- on the one hand they allow us to count situations of debit, lack or contrast with the negative numbers, and with the zero the absence or the balance of opposed.

When arise the division in parts of the unit, the previous numbers do not serve and the rational number must be created. This one must express two quantities, the number of parts in which the unit is divided (two parts, three, etc.) and how much of these parts I have. Here a complication appears to represent a fractional quantity; I need an integer number (numerator) and a natural number (denominator) that they join in the known representation of the fractions.

The arithmetic of these numbers is more complicated, since in order that the sum is possible, this one must be homogeneous, which means that we must have fractional units of the same kind (thirds, quarters, etc.). We will have therefore that to look for the fractional unit that all the addends share (the least common multiple).

The great problem arises when the number uses, not already to count, else to measure. Here the appearance of the continuous spatial one, from our macroscopic perspective, and the perfect measure pretension, these end in problems that lead to the irrational number.

In this way when it warns that the diagonal of a square can´t express it in terms of its side or any of its parts, for small that this one could be, and that the length of the circumference and the radius of this one have not a relation expressible as a fraction, a new type of number is created: *The irrational number,* that for definition is not expressible as

quotient of integer numbers. The immediate implication is that its decimal expression needs an infinite number of decimal not periodic figures since any number expressible with a quantity of finite or infinite periodic decimal figures can put as a quotient of integers.

We must been aware that they assume to the number, especially on having applied it to the measurement, traditionally divine properties: infinity and perfection (in the measurement).

But to measure is not a "divine" process on an ideal world and this is, I think, the origin of the quagmire in the mathematical description of the reality (apart from interpreting the contour -appearance- of this set that we are called A Universe as the essence of the same one). I explain this with an example: Let's suppose we are in the situation of the "inhabitants" of the Mandelbrot set border skilfully colored to show the incredible complexity and beauty of this border. On having observed it from inside we would manage to find patterns, laws that describe partially what we observe, but only if we should come out of what they show us our senses we would find the last essence of the set: a simple iterative process on the set of the complex numbers.

It should appear so, a new paradigm for the science; the simple observation and experimentation it is possible that never takes us to the compression of the Universe essence. Way might be depart from the necessary elements to create a simple structure and probably totally foreign of the reality appearance, that on having been developed on a numerical field it creates a few emergent properties of its application that they should generate the appearance of that we believe is the Universe. It is possible that the properties that we consider to be essential of reality (space, time, energy, movement ...) only be elements derived from others simple though secret to our experience.

To measure.

To measure is to establish a correspondence between the numbers and a scale in dimension or magnitude measured.

When we measure a length we establish a numerical correspondence with an scale defined on a (straight) rule. Every element of the scale defines an unit (chosen in agreement with the size of the measure and according to an standard pattern) and to every element a number is attributed departing from an end. This number is an approximation according to the precision of the measure pattern and the measurement realization.

In the measurement always it exists and to any scale an imprecision.

Zenon from Elea.

His birth places at the beginning of the 5th century BC.

Zenon demonstrated the logical absurdities from the plurality acceptance and the change as something real.

He supposes that the geometry dimension is constituted by more than one indivisible unit. In this case they will be possible two hypotheses:

First: the elementary unit is deprived of size

or, it comes

Second: that is real, and in the above mentioned case, will have a certain size.

For the first hypothesis we have that the above mentioned elements added to another element they would not make it greater and taken of another element they would not make it lower, therefore both operations would be void.

Since a geometric dimension constituted by void elements has to be likewise void.

In the second hypothesis it is possible to observe if this last unit is any thing real with a certain size, the dividing interval between two of any of them is real. But for the same reason the interval between this first dividing interval of any of these last two units in which it is included will be real also.

Thus and even the infinite, the multiple will contain infinite parts with a certain width and, in consequence, will turn out to be infinitely big.

The problem of classical geometry.

The classical geometry departs from undefinable concepts since it are the point and the straight line. Ideal both and without any real sense.

Let's think a bit like our mind might construct these abstract ideas.

The point. We observe a point in the real world when an object for its smallness or remoteness only impresses a cell of our retina.

In the visual field it had only information about position of this object. Not nevertheless, neither of size nor nature.

We establish a correspondence between visual information and mental abstraction. To the absence of size information and nature we make correspond the zero dimension and to the position information, inside the field of vision, we make correspond coordinates of position to the point in the plane. Plane that would be the infinite extension of the vision field.

Though this jump to the abstraction supposes a great intellectual advance, in view of our current knowledge also supposes the first step for a withdrawal of structure of the "reality".

The straight line. We say that something is straight, when we see it does not twist and remains in the same direction, since for example, the edge of a rectangular table.

In our habitual process of synthesis of the information, this border we simplify in the concept of straight line and prolong its characteristics and ends up to infinity. This way we return to separate of the "real" origin of our idea.

Both, straight line and point, we place them on the plane (infinite extension of the visual plane) creating the flat geometry that it is the only easier understood. The third dimension really we do not perceive it, but we observe a depth that result of parallax effect created by the eyeballs separation, in which they create flat images lightly different in every eye.

To perceive really the third dimension of an object would be to see it simultaneously from all the possible angles, in the same way as when we see a flat figure we see all his sides. In fact, only we take real conscience of the three-dimensional structure of an object when we move around it.

It seems to be deduced of the previous ideas, which to be able to perceive really a dimension it is necessary to observe, at least, from the immediate greater. We need the time to move in the space and to perceive the three-dimensionality of an object clearly.

The perception of the time.

Let's analyze our perception of the time:

- First let's see the time to very short period.

  If the attention is directed to the exterior, we perceive the course of the time for the change of position between the perceived objects.

  If the attention is directed towards our interior the change in our mental state is the reference to perceive the course of the time.

- In a long period, our memory is the one that helps us to order the events, they referenced some with regard to others, so that we locate them in a concrete position each one.

  This second way of perceiving the time is totally subjective and more vague than the previous one. Also it is less related to the " temporary dynamics " since the events are frozen like in this list that little by little is increased along our life.

  Only if we simulate mentally a past event, he acquires a " temporary dynamic " character.

In the measure of the time we use a dynamic periodic process, that is to say, the system in which the process takes place reaches, in a repeated way, the same state, and the separation between all the states repeated by that it passes, is the same.

It is therefore the referential variation of observable position what determines the form of our measurement of the time.

The natural of scale.

The last structure of the nature is countable and discrete.

The natural of scale is a discrete set of numbers that there takes the appearance of the real number but preserving the countability.

Why the natural of scale?. This nomenclature owes to that the definition is depending on the assumption of a measurement scale close to the human being but applicable in all its extension to the microscopic world.

Let's take an ordered and finite subset of natural numbers that departing from the one it is formed by sufficient elements to be applied to the measure, for example: 10^{200} that we will call $\mathbf{U}_0$ (universal cardinal) .$\mathbf{U}_0$ also it will represent to the set formed by all these numbers.

Let's take an intermediate value, for example: 10^{100} that determines the unit of scale **u**.

Let's establish now a correspondence between $\mathbf{U}_0$ and $\mathbb{Q}^+$ so that for everything r that belongs to $\mathbf{U}_0$ assigns an y belonging to s $\mathbb{Q}^+$ uch that

$$y = r \cdot \epsilon \quad \text{being} \quad \epsilon = \frac{1}{u} \quad \text{g}$$

so that the values of y they will express in decimal form.

To all the possible subsets of t $\mathbb{Q}^+$ his way obtained for different $\mathbf{U_0}$ and **u** we will call K ("κόσμος" →Cosmos) , cosmic numbers or natural of scale.

The unit only is fractionate in u parts and the minor part of the unit ϵ is. The minimal value for r is 1 and the maximum $\mathbf{U}_0$. Therefore, the minimal value for $y \in K$ will be its ϵ and maximum $\mathbf{U}_0 \cdot \epsilon$

Order in K.

The order in K for each $\mathbf{U_0}$ and **u** election, remains defined by the order in the natural numbers, we will say :

$$y > x$$

being $y = r' \cdot \epsilon$ and , $x = r \cdot \epsilon$ if:

$$r' > r$$

Operations in K.

ADDITION.

If we have $x, y \in K$, , then $x = r \cdot \epsilon \quad r \in \mathbb{N}$

$y = r' \cdot \epsilon \quad r' \in \mathbb{N}$

x and y sum is defined as:

$$\boxed{x + y = (r + r') \cdot \epsilon}$$

if $(r + r') < U_0$*.

Where $r + r'$ is the naturals conventional sum, $x + y$ therefore it belongs to K. This way, with the established restrictions, K is a closure under addition. Naturally the sum must be homogeneous because only is defined for elements of K that share the same ϵ . In the sum with conventional numbers the need of the homogeneity is not inserted in the own number nature, here nevertheless, having been tied ϵ to the type of observable measured the homogeneity is consubstantial to the process of the sum.

SUBTRACTION.

The difference between two numbers $x, y \in K$, if $x > y$ and $x = r \cdot \epsilon$, $y = r' \cdot \epsilon$

$$\boxed{x - y = (r - r') \cdot \epsilon}$$

(r-r') is the number that added to **r'** result **r**, since we are in the natural numbers.

This difference must not be understood as the sum of the additive inverse since it has not been defined this additive inverse element in K. As in the sum, this operation must be homogeneous therefore only it will be possible between elements of K that share the same ϵ .

* If U_0 represents the maximum value of any entity, the sum of the parts can never be major that the total.

Principles in K.

*Distinction principle in **K**.*

*Two numbers **x** and **y** belonging to **K** with,* $y > x$ *they are considered to be different if:*

$$y - x \geqslant 2\epsilon$$

in another case they will be indistinguishable and exchangeable and will be represented

$$x \diamond y$$

Uncertainty principle.

*Any number **x** belonging to **K** is indeterminate in a neighborhood of radius* ϵ .

This type of number will apply to measure physical observables that, in last instance, they will be quantized and will be, its values, countable. For every observable it will be done the choice of $\mathbf{U_0}$ and **u** appropriate.

Addition properties.

Associative property

If $x, y, z \in K$, them $x = r \cdot \epsilon$, $y = r' \cdot \epsilon$, $z = r'' \cdot \epsilon$

$$(x + y) + z = [(r + r') \cdot \epsilon] + z = [(r + r') + r''] \cdot \epsilon$$

and for the associative of naturals

$$[(r + r') + r''] \cdot \epsilon = [r + (r' + r'')] \cdot \epsilon = x + [(r' + r'') \cdot \epsilon] = x + (y + z)$$

and them:

$$(x + y) + z = x + (y + z)$$

Identity element.

Isn't a task easy to define the identity element and this problem, we are going to see it is related to the uniqueness break.

In the first of two principles previously proposed is course given to think about ϵ as identity element.

Be two numbers $x, y \in K$, with . $y > x$

–If , $y - x < 2\epsilon \Rightarrow x \diamond y$ or indistinguishable for the distinction principle, but if

$$y - x < 2\epsilon \Rightarrow y - x = \epsilon$$

or

$$y = x + \epsilon$$

but how $x \diamond y$, them

$$x \diamond x + \epsilon$$

But there appears the indistinguishability of all the numbers, so if

$$x \diamond x + \epsilon = r \cdot \epsilon + 1 \cdot \epsilon = (r+1) \cdot \epsilon$$

and how

$$x = r \cdot \epsilon$$

Then any number is indistinguishable of the later (or previous one), this way, it seems that only we would take ϵ as a number with own identity since all the rest would be indistinguishable.

The indistinguishability between all the numbers resolves across the distinction principle since though a number would be indistinguishable of immediately later or previous, it would not be from the following ones to these, since if we have the numbers

$$x\,,\ x+\epsilon\ ,\ x+2\epsilon$$

then as we have seen

$$x \diamond x + \epsilon \quad \text{and} \quad x + \epsilon \diamond x + 2\epsilon$$

but this relation isn't transitive since x isn't indistinguishable of $x + 2\epsilon$ since

$$x + 2\epsilon - x = (r+2) \cdot \epsilon - r \cdot \epsilon = 2\epsilon$$

and for the distinction principle they are different or not exchangeable.

In ***K*** two numbers can be different but indistinguishable.

Let's take two numbers x e $y \in \boldsymbol{K}$, that since we have just said that they are two numbers, cannot be the same. Then

if $x \diamond y$ and $x < y \ \Rightarrow \ y - x < 2\epsilon \ \Rightarrow \ y - x = \epsilon$; $y = x + \epsilon$ according to this y is the following to x.

if $y < x \ \Rightarrow \ x - y < 2\epsilon \ \Rightarrow \ x - y = \epsilon$; $x = y + \epsilon$ and according to this x is the following to y.

If only we know that two numbers are indistinguishable, or the first is the following to the second one or on the contrary. And this will be the habitual situation since the numbers are assigned to results of measures or are assigned hypothetically to values of observable, we do not know *a priori* its exact value only we know if they are distinguishable or not of others. And if they are not distinguishable, in no case we will know if they are equal or not, or which of them is the biggest.

On the one hand ϵ is the minimal value in which the number can be increased and simultaneously is the " identity element " of the sum though in a very special way (in relation only with the indistinguishability).

The distinguishability of a number and its immediate later or previous is not possible, so these numbers are blurred at its minimal scale.

According to my comments about the break of the uniqueness, these numbers allow the break above mentioned, since if $\epsilon \diamond 2\epsilon$, $2\epsilon \diamond 3\epsilon$ and so on, the numeral of an entity might vary from number one to any value in a spontaneous way without entering in a conflict with the nature of these numbers.

Additive inverse.

The existence of additive inverse for numbers that will be used to measure, implies that the observable object of the measure must have two opposite facets. For example, the electrical charge is an observable that has two opposite manifestations, one is the electron charge and other one the proton charge, which in quantity are equal but its character is opposite.

Deliberately, hitherto, I have omitted the introduction of the zero. The zero will not be a result of a measure but the representative of a situation of balance between opposite values of an observable.

Any measure will have always a minimal value ϵ since it will be, though the measure device is so perfect as is possible, indeterminate in this value.

If we take these premises into account, we can say:

1.-Zero is not the result of a measure, which implies that $0 \notin \boldsymbol{K}$.

2.-Zero only represents a situation of balance.

3.-Zero cannot be summed, because only the addition of observable values is possible.

Can already think about the definition of additive inverse in $\boldsymbol{K}$.

If ϵ is minimal value of an observable in $\boldsymbol{K}$ and this observable has two opposite facets, we can designate them like $+\epsilon$ and $-\epsilon$, so $-y = r\cdot(-\epsilon)$ with $r \in \mathbb{N}$.

We will call zero the sum of $+\epsilon$ and $-\epsilon$ this will represent a situation of balance between opposed elements not absence of these.

Then additive inverse of a $x \in \boldsymbol{K}$ it is defined as the one added to x gives zero.

$$x+(-x)=0 \quad \Rightarrow \quad r\cdot\epsilon + r\cdot(-\epsilon) = r\cdot\{\epsilon+(-\epsilon)\} = r\cdot 0 = 0$$

This sum is not homogeneous because though are summed observables of the same type they have opposite facets.

In this case also the result does not belong to $\boldsymbol{K}$ and the sum is not a closure under addition either.

We will bear in mind also that are conceptual different the subtraction how we have defined previously and the sum of additive inverse, though the results can be equal. Also only it will be necessary to speak about additive inverse when the observable object of measure has opposite characters.

NATURAL MULTIPLICATION.

The natural multiplication is an operation between a natural number and a number belonging to $\boldsymbol{K}$.

The natural indicates the number of times that has to add the number belonging to $\boldsymbol{K}$.

If $a \in \mathbb{N}$ and $x \in K$

$$a \cdot x = x \cdot a = \underbrace{x + x + x \cdots + x}_{a-\text{times}}$$

$$a \cdot x = (a \cdot r) \cdot \epsilon$$

MULTIPLICATION IN *K.*

We must not forget that the belonging numbers to $\boldsymbol{K}$ are observable values .

If numbers to multiply represent values of the same kind of observable, each of ϵ them it will be of the same kind and we will have to ask the product meaning of two values of the same observable, that is to say, of $\epsilon \cdot \epsilon$ .This case might associate to a dimensionality change in the observable,being modified $\mathbf{U_0}$ and $\mathbf{u}$.

If numbers to multiply represent different observable we will have to ask the product meaning as new observable.

I would like to insist on the absolute need to understand before its utilization the meaning of above mentioned products and its opportunity.

Multiplication properties.

Strictly speaking, not at least in $\mathbb{R}$, when we use the numbers to measure, the

product is a closure operation. For example, if we multiply two numbers that represent measures of length, the result represents a surface measure. Therefore, the nature of the product is different to the nature of the factors.

We can wait, that in numbers created for the physics, these considerations are inserted in its own nature.

Be two numbers $x, y \in K$ that $x = r \cdot \epsilon$ and $y = r' \cdot \epsilon'$ we call multiplication $x \cdot y$ to:

$$x \cdot y = r r' \cdot (\epsilon \epsilon')$$

that it is a **closure operation** because the result has the structure of a number belonging to K.

If we call $\delta = \epsilon \epsilon'$, δ defines a new blurry scale in the magnitude products.

It turns out to be evident the associativity so much as the commutativity since $r r'$ is a natural product and $\epsilon \epsilon'$ a "rational" product.

Identity element.

The identity element in multiplication will be $1 \in K$, that is, $1 = u \cdot \epsilon$ or $1 = u' \cdot \epsilon'$. This way: $x \cdot 1 = r u \cdot \epsilon^2$ or $x \cdot 1 = r u' \cdot \epsilon \epsilon'$ that though it has naturally a different structure from x, since it happens in any product, has x the same value. We can put therefore:

$$x \cdot 1 = x$$

Inverse element.

The inverse element of x will be an $x^* \in K$ such that:

$$x \cdot x^* = 1$$

Let's see that not always inverse exists:

Let's suppose that $x = r \cdot \epsilon$ for $\mathbf{U_o}$ and **u** correspondents to the observable that we use as the first factor and that the 1, for the product of observable that we are making, obtains from $\mathbf{U_o''}$ and **u''** how $1 = u'' \cdot \epsilon''$, being $x^* = n \cdot \delta$ with $\mathbf{U_o'}$ and **u'** for the second factor. It will have to be thereforet $n = \frac{u''}{r}$ hat should belong to $\mathbb{N}$ but it does not in all the cases since the quotient of two naturals is not always a natural.

Therefore inverse element does not exist for any x.

NATURAL DIVISION.

If $a \in \mathbb{N}$ and $x \in K$, it is defined as a natural division in K, x over a :

$$x/a$$

how a number $y \in \mathbf{K}$ such that $y \cdot a$ belonging to neighborhood centered in x and radius $(a-1)\cdot\epsilon$

So, if $x = r \cdot \epsilon$ and $x = r \cdot \epsilon$

$$x/a = (r/a)\cdot\epsilon$$

For example: If $\mathbf{U_0} = 10000$, $\mathbf{u} = 100$ and $\epsilon = \frac{1}{100}$ then:

$$83{,}23/4 = (8323/4)\cdot\epsilon = 2080\cdot\epsilon = 20{,}80$$

It would also be valid the solution 20,81, since it would be inside the neighborhood centered in x and radius $3\cdot\epsilon$ but this would not be any problem since according to distinction principle they are indistinguishable.

K extension. Numerical cycle-bases.

Hitherto I have created a number that supports, in its conventional use for the mesoscopic* and macroscopic physics, the appearance of the real number, in the above mentioned media, the used numbers have a decimal expansion limited to the measure precision, which is generally very far from the value of ϵ correspondent to observable measured. Nevertheless, when we move in the quantum physics these numbers they correspond better with the nature structure at this scale.

As the real number was extended with the complex numbers to open the range of applicability, also we will do it with K.

Always I studied the complex numbers, the justification of imaginary number i turned out to be unconvincing to me. I was thinking that this one, had to belong to a structure that was spreading, for saying it somehow, for below and over it and it was not related to the quaternions, octonions not none of other structures created to extend the complex numbers. From this search arises the concept of numerical cycle - base .

* I distinguish three scales in nature: microscopic in which make patents the effects of quantum mechanics, mesoscopic from previous up some light years and macroscopic from the previous to the size of the universe.

CYCLE-BASE	NUMBER	POWERS	PERIOD	EQUIVALENCES	POWERS equivalents
First	**g**	$g^1=g$ $g^2=g$	**T=1** (2^0)	**g =1=g**	$g^1=1$ $g^2=1$
Second	**h**	$h^1=gh$ $h^2=g$ $h^3=gh$	**T=2** (2^1)	**gh=h= -1**	$h^1=-1$ $h^2=1$ $h^3=-1$
Third	**i**	$i^1=gi$ $i^2=gh$ $i^3=ghi$ $i^4=g$ $i^5=gi$	**T=4** (2^2)	**ghi=-i**	$i^1=i$ $i^2=-1$ $i^3=-i$ $i^4=1$ $i^5=i$
Fourth	**j**	$j^1=gj$ $j^2=gi$ $j^3=gij$ $j^4=gh$ $j^5=ghj$ $j^6=ghi$ $j^7=ghij$ $j^8=g$ $j^9=gj$	**T=8** (2^3)	**ghij=-j**	$j^1=j$ $j^2=i$ $j^3=j$ $j^4=-1$ $j^5=-j$ $j^6=-i$ $j^7=-j$ $j^8=1$ $j^9=j$
Fifth	**k**	$k^1=gk$ $k^2=gj$ $k^3=gjk$ $k^4=gi$ $k^5=gik$ $k^6=gij$ $k^7=gijk$ $k^8=gh$ $k^9=ghk$ $k^{10}=ghj$ $k^{11}=ghjk$ $k^{12}=ghi$ $k^{13}=ghik$ $k^{14}=ghij$ $k^{15}=ghijk$ $k^{16}=g$ $k^{17}=gk$	**T=16** (2^4)	**ghijk=-k**	$k^1=k$ $k^2=j$ $k^3=k$ $k^4=i$ $k^5=k$ $k^6=j$ $k^7=k$ $k^8=-1$ $k^9=-k$ $k^{10}=-j$ $k^{11}=-k$ $k^{12}=-i$ $k^{13}=-k$ $k^{14}=-j$ $k^{15}=-k$ $k^{16}=1$ $k^{17}=k$
.	.	.	.	.	.
.	.	.	.	.	.

MULTIPLICATION TABLE FOR FIRST FIVE CYCLE-BASES

•	g	h	i	j	k
g	g	h	i	j	k
h	h	g	-i	-j	-k
i	i	-i	h	j	k
j	j	-j	j	i	k
k	k	-k	k	k	j

Numerical sets construction.

Combining the belonging numbers to K with each of the cycle-bases obtain different numerical sets.

If we combine g with K considering only values $x \in K$ that should correspond with positives observables there would be obtained what we might call K^+.

If we combine h with K considering only values $x \in K$ that should correspond with negatives observables there would be obtained what we might call K^-.

From the union of both previous combinations with the sum, we would obtain the numbers adapted to represent observables with values opposed:

$$x_0 g + x_1 h$$

where x_0 it would be the observable value with positive character and x_1 the value observable with negative character, whereas the above mentioned characters (positive and negative) determine it g and h.

If we combine with K the cycle-base g, h and i, we will obtain what we can be call the cosmic complex numbers of order two, which we will denote $K\mathbb{C}2$. Where 2 it refers to the exponent of two who determines the period of the cycle-base. This way a $x \in K\mathbb{C}2$ would express:

$$x = x_0 g + x_1 h + x_2 i$$

If we combine with K the cycle-bases g, h, i and j we will obtain what we can be call the cosmic complex numbers of order three, which we will denote $K\mathbb{C}3$. Where 3 it refers to the exponent of two who determines the period of the cycle-base. This way a $x \in K\mathbb{C}3$ would express:

$$x = x_0 g + x_1 h + x_2 i + x_3 j$$

Of equal way we would operate for following cycle-bases combined with K.

$K\mathbb{C}3$ STRUCTURE.

If $x \in K\mathbb{C}3$ we can represent x how:

1. $x = x_0 g + x_1 h + x_2 i + x_3 j$ with $x_i \in K$.
2. How a vector $(x_0 - x_1, x_2, x_3)$ with the base $\{g, i, j\}$.
3. How 4-tuple . (x_0, x_1, x_2, x_3)

ADDITION

If we have $x, y \in K\mathbb{C}3$ is defined as addition $x+y$:

1. $x+y=(x_0+y_0)g+(x_1+y_1)h+(x_2+y_2)i+(x_3+y_3)j$
2. $(x_0-x_1, x_2, x_3)+(y_0-y_1, y_2, y_3)=((x_0+y_0)-(x_1+y_1), x_2+y_2, x_3+y_3)$
3. $(x_0, x_1, x_2, x_3)+(y_0, y_1, y_2, y_3)=(x_0+y_0, x_1+y_1, x_2+y_2, x_3+y_3)$

Properties:

1. Addition is a closure operation (obvious).
2. It is associative, because addition is associative in K.
3. The identity element is ϵ the K identity element.
4. Opposite element for x is xh. (h changes the observable nature to its opposite character, h would be, for example, the operator who changes the electrical positive charge to negative and vice-verse).

   Let's verify:

   If we have $x=x_0g+x_1h+x_2i+x_3j$ let's see the value of $x+xh$

   $$xh=x_0gh+x_1h^2+x_2ih+x_3jh=x_0h+x_1g-x_2i-x_3j$$

   $$x+xh=(x_0g+x_1h+x_2i+x_3j)+(x_0h+x_1g-x_2i-x_3j)=$$
   $$=(x_0+x_1)g+(x_1+x_0)h+(x_2-x_2)i+(x_3-x_3)j=0$$

   since g=1 and h= -1. (It understands the zero in the sense defined in K).

5. The sum is commutative, for it being the sum in K.

   Addition in $K\mathbb{C}3$ has abelian group structure in sense of K.

MULTIPLICATION

It is defined the multiplication in $K\mathbb{C}3$ as:

$$x\cdot y=(x_0g+x_1h+x_2i+x_3h)\cdot(y_0g+y_1h+y_2i+y_3j)=$$

$$=(x_0y_0+x_1y_1)g+(x_0y_1+x_1y_0+x_2y_2)h+$$
$$+(x_0y_2-x_1y_2+x_2y_0-x_2y_1+x_3y_3)i+$$
$$+(x_0y_3-x_1y_3+x_2y_3+x_3y_0-x_3y_1+x_3y_2)j$$

though it is more suitable to use the following matrix notation:

$$\boldsymbol{x}\cdot\boldsymbol{y}=(x_0,x_1,x_2,x_3)\begin{pmatrix}y_0&0&0&0\\0&y_1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\boldsymbol{g}+(x_0,x_1,x_2,x_3)\begin{pmatrix}0&y_1&0&0\\y_0&0&0&0\\0&0&y_2&0\\0&0&0&0\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\boldsymbol{h}+$$
$$+(x_0,x_1,x_2,x_3)\begin{pmatrix}0&0&y_2&0\\0&0&-y_2&0\\y_0&-y_1&0&0\\0&0&0&y_3\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\boldsymbol{i}+(x_0,x_1,x_2,x_3)\begin{pmatrix}0&0&0&y_3\\0&0&0&-y_3\\0&0&0&y_3\\y_0&-y_1&y_2&0\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\boldsymbol{j}$$

Properties:

1. Since we can see across definition is a closure operation.
2. Let's verify the associativity:

$$\begin{aligned}(\boldsymbol{x}\cdot\boldsymbol{y})\cdot z=&(x_0y_0z_0+x_1y_1z_0+x_0y_1z_1+x_1y_0z_1+x_2y_2z_1)\boldsymbol{g}+\\&+(x_0y_1z_0+x_1y_0z_0+x_2y_2z_0+x_0y_0z_1+x_1y_1z_1)\boldsymbol{h}+\\&+\big(x_2y_0z_0-x_2y_1z_0+x_0y_0z_0-x_1y_2z_0+x_3y_3z_0-x_2y_0z_1+x_2y_1z_1-x_0y_0z_1+x_1y_2z_1-\\&-x_3y_3z_1+x_0y_0z_2+x_1y_1z_2-x_0y_1z_2-x_1y_0z_2-x_2y_2z_2+x_3y_0z_3-x_3y_1z_3+x_3y_2z_3+\\&+x_0y_3z_3-x_1y_3z_3+x_2y_3z_3\big)\boldsymbol{i}+\\&+\big(x_3y_0z_0-x_3y_1z_0+x_3y_2z_0+x_0y_3z_0-x_1y_3z_0+x_2y_3z_0-x_3y_0z_1+x_3y_1z_1-x_3y_2z_1-\\&-x_0y_3z_1+x_1y_3z_1-x_2y_3z_1+x_3y_0z_2-x_3y_1z_2+x_3y_2z_2+x_0y_3z_2-x_1y_3z_2+x_2y_3z_2+\\&+x_0y_0z_3+x_1y_1z_3-x_0y_1z_3-x_1y_0z_3-x_2y_2z_3+x_2y_0z_3-x_2y_1z_3+x_0y_0z_3-x_1y_2z_3+\\&+x_3y_3z_3\big)\boldsymbol{j}\end{aligned}$$

,

$$\begin{aligned}\boldsymbol{x}\cdot(y\cdot z)=&(x_0y_0z_0+x_1y_1z_0+x_0y_1z_1+x_1y_0z_1+\underline{x_1y_2z_2})\boldsymbol{g}+\\&+(x_0y_1z_0+x_1y_0z_0+\underline{x_0y_2z_2}+x_0y_0z_1+x_1y_1z_1)\boldsymbol{h}+\\&+\big(x_2y_0z_0-x_2y_1z_0+\underline{x_0y_2z_2}-x_1y_2z_0+x_3y_3z_0-\underline{x_0y_2z_1}+x_2y_1z_1-x_0y_0z_1+x_1y_2z_1-\\&-x_3y_3z_1+x_0y_0z_2+x_1y_1z_2-x_0y_1z_2-x_1y_0z_2-x_2y_2z_2+x_3y_0z_3-x_3y_1z_3+x_3y_2z_3+\\&+x_0y_3z_3-x_1y_3z_3+\underline{x_3y_3z_2}\big)\boldsymbol{i}+\\&+\big(x_3y_0z_0-x_3y_1z_0+\underline{x_3y_2z_2}+x_0y_3z_0-x_1y_3z_0+x_2y_3z_0-x_3y_0z_1+x_3y_1z_1-x_3y_2z_1-\\&-x_0y_3z_1+x_1y_3z_1-x_2y_3z_1+x_3y_0z_2-\underline{x_3y_2z_2}+x_0y_3z_2-x_1y_3z_2+x_2y_3z_2+\\&+x_0y_0z_3+x_1y_1z_3-x_0y_1z_3-x_1y_0z_3+\underline{x_2y_2z_3}+x_2y_0z_3-x_2y_1z_3+\underline{x_0y_2z_3}-x_1y_2z_3+\\&+x_3y_3z_3\big)\boldsymbol{j}\end{aligned}$$

   the marked terms do not coincide therefore the associative property is not fulfilled.
3. Commutative property:

$$\begin{aligned}\boldsymbol{x}\cdot\boldsymbol{y}=&(x_0y_0+x_1y_1)\boldsymbol{g}+(x_0y_1+x_1y_0+x_2y_2)\boldsymbol{h}+\\&+(x_0y_2-x_1y_2+x_2y_0-x_2y_1+x_3y_3)\boldsymbol{i}+\\&+(x_0y_3-x_1y_3+x_2y_3+x_3y_0-x_3y_1+x_3y_2)\boldsymbol{j}\end{aligned}$$

$$\begin{aligned}\boldsymbol{y}\cdot\boldsymbol{x}=&(y_0x_0+y_1x_1)\boldsymbol{g}+(y_1x_0+y_0x_1+y_2x_2)\boldsymbol{h}+\\&+(y_2x_0-y_2x_1+y_0x_2-y_1x_2+y_3x_3)\boldsymbol{i}+\\&+(y_3x_0-y_3x_1+y_3x_2+y_0x_3-y_1x_3+y_2x_3)\boldsymbol{j}\end{aligned}$$

   And bearing in mind the commutativity in $\boldsymbol{K}$ we see that the product in $\boldsymbol{K}\mathbb{C}\boldsymbol{3}$ is commutative.

4. Identity element.

   Evidently $\boldsymbol{g}$ is the product identity element, but not the only one. Let's see:

   If it exist an $\boldsymbol{x'}$ identity element of the multiplication then $\boldsymbol{x \cdot x' = x}$.

$$\begin{aligned} x\cdot x' &= (x_0 g + x_1 h + x_2 i + x_3 j)\cdot(x'_0 g + x'_1 h + x'_2 i + x'_3 j) = (x_0 x'_0 + x_1 x'_1) g + \\ &+ (x_1 x'_0 + x_0 x'_1 + x_2 x'_2) h + (x_2 x'_0 - x_2 x'_1 + x_0 x'_2 - x_1 x'_2 + x_3 x'_3) i + \\ &+ (x_3 x'_0 - x_3 x'_1 + x_3 x'_2 + x_0 x'_3 - x_1 x'_3 + x_2 x'_3) j = x_0 g + x_1 h + x_2 i + x_3 j. \end{aligned}$$

   Wherefrom we obtain the following system:

$$\begin{cases} x_0 = x_0 x'_0 + x_1 x'_1 \\ x_1 = x_1 x'_0 + x_0 x'_1 + x_2 x'_2 \\ x_2 = x_2 x'_0 - x_2 x'_1 + x_0 x'_2 - x_1 x'_2 + x_3 x'_3 \\ x_3 = x_3 x'_0 - x_3 x'_1 + x_3 x'_2 + x_0 x'_3 - x_1 x'_3 + x_2 x'_3 \end{cases}$$

   And grouping the unknowns:

$$\begin{cases} x_0 = x_0 x'_0 + x_1 x'_1 \\ x_1 = x_1 x'_0 + x_0 x'_1 + x_2 x'_2 \\ x_2 = x_2 x'_0 - x_2 x'_1 + (x_0 - x_1) x'_2 + x_3 x'_3 \\ x_3 = x_3 x'_0 - x_3 x'_1 + x_3 x'_2 + (x_0 - x_1 + x_2) x'_3 \end{cases}$$

   The range of this system depends only on the coefficients matrix since the extended matrix only adds an identical column to the first one in the coefficients matrix . If we study the range of the above mentioned matrix:

$$\begin{vmatrix} x_0 & x_1 & 0 & 0 \\ x_1 & x_0 & x_2 & 0 \\ x_2 & -x_2 & (x_0 - x_1) & x_3 \\ x_3 & -x_3 & x_3 & (x_0 - x_1 + x_2) \end{vmatrix} = (x_0 + x_1) \begin{vmatrix} (x_1 - x_0) & x_2 & 0 \\ -x_2 & (x_0 - x_1) & x_3 \\ -x_3 & x_3 & (x_0 - x_1 + x_2) \end{vmatrix}$$

   we see that the range will be lower than the number of unknowns for $x_0 = -x_1$ and the solution will not be only one. Therefore, the identity element it is not either.

5. Distributive property.

   It is easy to see that the product is distributive respect the sum.

CHAPTER II

PREMETRIC.

Hitherto we have defined some basic operations, we will start by studying some concepts before the metrics that the natural of scale uses on.

Before nothing we will define as **neighborhood of minimal lack of definition 2ϵ** .

The Reality.

The reality is a creation subjective and shared between the observers, consisted in form and scale in which they perceive the observables that are to his scope.

To clarify the meaning of the previous definition let's see an example:

The spatial reality. An observer perceives the space as a separation between entities. And this way it is perceived by all the observers who share a similar structure of information processing. If the observer is a human being, all the human beings who share naturally above mentioned structures of processing, they will perceive it equally, will have a common experience.

The observer obtains values of position observable of the entities in his environment, values that does not shares neither with them nor between them and does it through the mechanisms of interaction that more or less we know.

It is not that they necessarily objects and observer have to occupy a different place or even that have to occupy some place, that is to say, that the sensation is the objective form of reality, but it is the appearance, the way in which the observer assimilates the observable.

The perception of the reality is constructed across the interaction, which is the form in which the entities in all the levels of complexity take "conscience" of other entities and his own identity.

In order that it could exist a reality must happen a break of the uniqueness in multiple entities capable of possessing different observable values.

In order that a reality could be perceived must also exist mechanisms of interaction of the entities, which are somehow related to the values of the observable. It is the interaction what supports the break of uniqueness .

The levels of consciousness.

Although, at first, the term consciousness is only attributed to a human capacity, we will extend its application to any being who somehow modifies its state or some kind of internal self representation by the presence of another, with whom it interacts.

This way we might speak about the most elementary form of consciousness, which would be the mere interaction between a particle and other one across the fundamental forces. This is the consciousness of all those beings who do not possess a integration system of sensory information. The activity that expresses the highest level inside this form would be the codified transmission, by means of genes, from the adaptation mechanisms to environment.

The following level of consciousness we might name it “instinctive conscience”. The element of the complexity level capable of acquiring it will have a integration system of sensory information. Its expression takes place across automatic, predefined conducts in the structures of the integration system of information of the individual, developing - the conducts - for the stimuli concurrence or trigger sensations.

When the repertoire of automatic conducts becomes so extensive and complex that a group of stimuli could trigger various patterns of conducts, is required the emergence of a new system for the integration of the different patterns allowing a choice. This system takes decisions from an internal state of sensations provoked by the results produced by previous choices. If a previous choice produced a failure be eliminate because they produce internal tensions that displease the individual. We are at the level of sentimental consciousness.

In its more elaborate form, feelings, are able to even induce pathologies psycho-physical response to adverse situations, as well as general well-being while the environment is hostile. The alive beings in his fight for the environment adaptation , little by little are developing in

the support of the sentimental consciousness, structures capable of storing information in a more effective way.

On the other hand, there comes a moment in which the feelings are insufficient for the problems resolution that they present to some species with structural possibilities of environment manipulation , here there can appear a process that we would identify with the intuition, capable of logical - deductive reasoning probably of parallel type, together with a new system of data processing based on logical - deductive sequential processes, processes that take as basis primary, data contained in the memory, determining these the results with the subjective character that it implies. We then have an internal representation of reality.

When the memory reaches a high level and the reason develops to a certain extent can appear the self- consciousness and the transcendent thought. The individual, conscious itself, questions his paper in the world , as well as the essence and paper of this world in the existence. This activity is the one that demand of the conscious beings a more intense exercise of the reason, which controls his aims, as well as of the intuition.

None of the species naturally leaves forms of consciousness of the previous levels but they are still developing a very important role.

The natural principles.

Understand as natural principles those that concern the definition needs of the entities (identification) and are the necessary bases for the construction of a universe. They have an arbitrary character and are cognizable but not justifiable, except for its need to obtain the structures that we observe.

Some definitions.

Magnitude or observable. Perceptive aspect of a reality to which it is possible to attribute numbers

Dimension. Each different numbers that are assigned to a magnitude for its measure.

Observable dimension. Number of numbers that are assigned to an observable.

Point of dimension N. In order that a break of the uniqueness could exist, this one must be disintegrated in entities that do not share the value of any N dimension observable . If observable saying possesses minimal value ϵ in all its dimensions it is said that it is a punctual observable of dimension N or N dimension point.

To measure. To look for the set of numbers that defines an observable of an entity .

Particle. Particle or elementary entity is the observables complete set associate to an element of the uniqueness break.

This way the observable size of a point or particle is the set of numbers that determine the lack of definition of the above mentioned entity. On the other hand, observable position will be the set of numbers that there breaks the uniqueness of the point or particle.

- *The observables names neither define nor attribute any intrinsic property of the entity, point or particle. Its only relation with the observable is that of our perception of it, that is to say, the observables do not possess any category, concept..., these belong only to the observable perception. Therefore, when I speak about size, this category of the observable belongs to the perception of it, but it is not the observable itself.*
- *The entities identity (identification) remains determined by the values of its observables and the identity of these (the observables) for the types of interaction and they are to these, to which we attribute categories, since the interactions are those that determine the observable perception and the structure of our reality.*

At this moment of our analysis we might venture a definition of number as:

NUMBER. Plausible describer of the structure of the Absolute cognizable.

And also:

WORD. Synthetic, subjective and arbitrary describer of the perceptions.

N-space or N dimension space. Area of total lack of definition in N dimensions in that there can remain totally "located" a point of N-dimension once fixed a frame of reference.

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