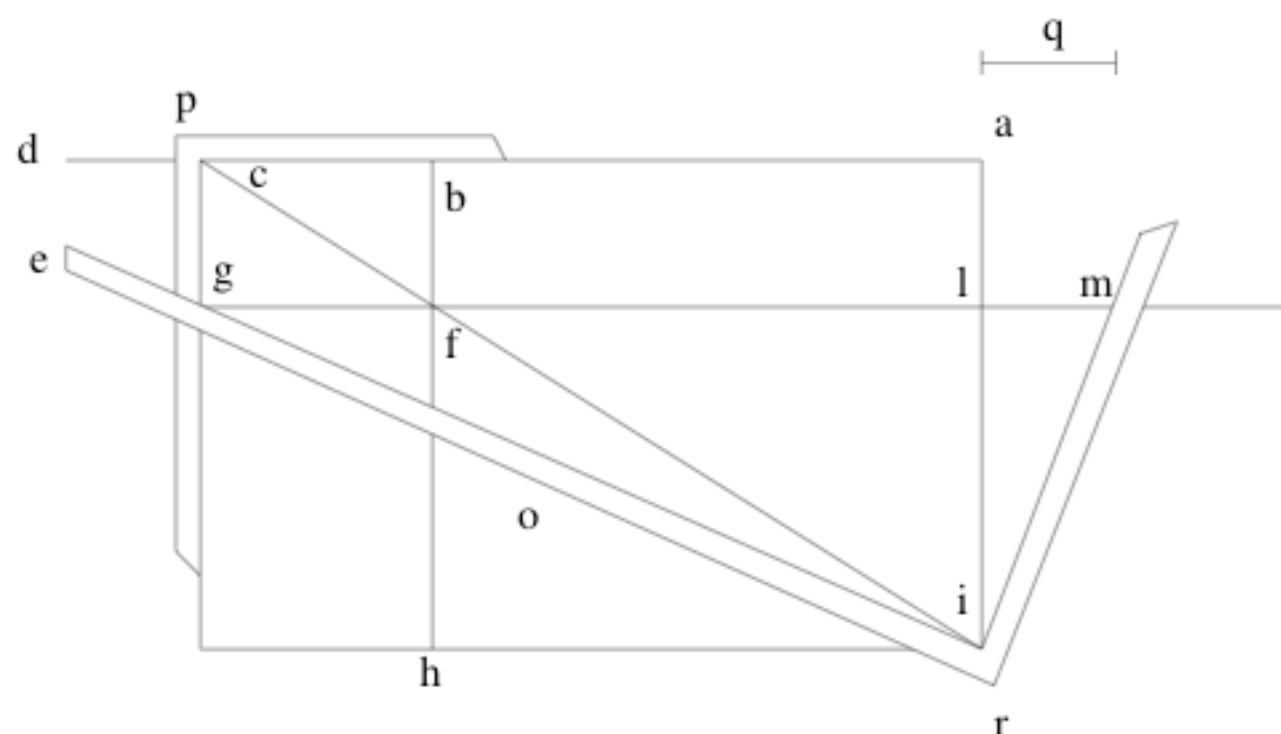


i.e., Cardano's formula for the solution $x = t - \frac{p}{3t}$ of the cubic equations of the form $x^3 + px = q$. All this is performable geometrically to actually produce the x only if t^3 is a real number. That is, this geometric demonstration doesn't work in the irreducible case¹⁴.

The method in the plane.

Bombelli's second method resembles some of the *neusis*-constructions in ancient Greek geometry—used in questions of angle-trisection (see § 5 below) and indeed does work in the irreducible case. Bombelli promotes this method (invoking of the august authority of the ancient authors, who used similar methods) because, he claims, it provides a “geometric demonstration” that his cubic radicals “exist”.

By a *gnomon* let us mean an “L-shaped” figure; i.e., two closed line segments joined at a 90 degree angle at their common point (the *vertex*). Bombelli uses a construction with two gnomons (if that is the plural form of the word), one with vertex r and one with vertex unfortunately labeled p in the diagram (taken from his manuscript) below.



He will construct such a diagram from the data of his cubic equation $x^3 = px + q$, i.e., from the pair of real numbers p and q (from dimension considerations, we can expect p to appear as an area, and q/p as a linear measurement). Let us calibrate the diagram by putting

$$\overline{lm} = \text{unity}.$$

¹⁴ This type of “decomposition of the cube” argument had already been used by Cardano in the *Ars Magna* to explain how, for a particular equation ($x^6 + 6x = 20$) one can derive his formula; Cardano never considered the irreducible case.